Finite Math - Fall 2016

Lecture Notes - 11/14/2016

Section 7.2 - Sets

Set Properties and Notation. A *set* is a collection of objects specified in such a way that we can tell whether any given object is in the set. We usually will denote sets by capital letters. Objects in a set are called *elements* or *members* of the set. Symbolically,

$a \in A$ means "a is an element of the set A"
$a \notin A$ means "a is not an element of the set A"

It is possible to have a set without any elements in it. We call this set the *empty set* or *null set*. We denote this set by $\varnothing$. An example of a set which is empty is the set of all people who have been to Mars.

We often denote sets by listing their elements between a pair of braces: $\{\ \}$. For example, the following are sets:

$$\{0, 1, 2, 3, 4, 5\}, \{a, b, c, d, e\}, \{1, 2, 3, 4, 5, ...\}.$$

Another common way to write sets is by writing a rule in between braces. For example,

$$\{x|x \text{ is even}\}, \{x|x \text{ is a traditional Chinese element}\}, \{z|z^2 = 1\}.$$

The way to read this second type of set is, for example, "the set of x such that x is even" or "the set of z such that $z^2 = 1$. Notice that we get two kinds of sets like this: *finite sets* (the set only has finitely many elements) and *infinite sets* (the set has infinitely many elements). The sets $\{1, 2, 3, 4, 5...\}$ and $\{x|x \text{ is even}\}$ are infinite sets while the others are finite.

Example 1. *Let G be the set of all numbers whose square is* 9.

(a) *Denote G by writing a set with a rule (the second style above).*

(b) *Denote G by listing the elements (the first style above).*

(c) *Indicate whether the following are true or false:* $3 \in G$, $9 \in G$, $-3 \notin G$.

Solution.

(a) $\{x|x^2 = 9\}$

(b) $\{-3, 3\}$

(c) *True, False, False*

Suppose we have two sets A and B. If every element in the set A is also in the set B, we say that A is a *subset* of B. By definition, every set is a subset of itself. If A and B

have the exact same elements, then we say the sets are *equal.* Here is some notation for this:

$A \subset B$	means	"A is a subset of the set B"
$A \not\subset B$	means	"A is not a subset of the set B"
$A = B$	means	"A and B have the exact same elements"
$A \neq B$	means	"A and B do not have the exact same elements"

It follows that $\varnothing$ is a subset of every set and if $A \subset B$ and $B \subset A$, then $A = B$.

Example 2. *Let* $A = \{-3, -1, 1, 3\}$, $B = \{3, -3, 1, -1\}$, $C = \{-3, -2, -1, 0, 1, 2, 3\}$. *Decide the truth of the following statements*

$$\begin{array}{lll} A = B & A \subset C & A \subset B \\ C \neq A & C \not\subset A & B \subset A \\ \varnothing \subset A & \varnothing \subset C & \varnothing \notin A \end{array}$$

Solution. *All true.*

Example 3. *Let* $A = \{0, 2, 4, 6\}$, $B = \{0, 1, 2, 3, 4, 5, 6\}$, $C = \{2, 6, 0, 4\}$. *Decide the truth of the following statements*

$$\begin{array}{lll} A \subset B & A \subset C & A = C \\ C \subset B & B \not\subset A & \varnothing \subset B \\ 0 \in C & A \notin B & B \subset C \end{array}$$

Solution. *All true except for* $B \subset C$.

Example 4. *Find all subsets of the following sets:*

(a) $\{a, b\}$

(b) $\{1, 2, 3\}$

(c) $\{\alpha, \beta, \gamma, \delta\}$

Solution.

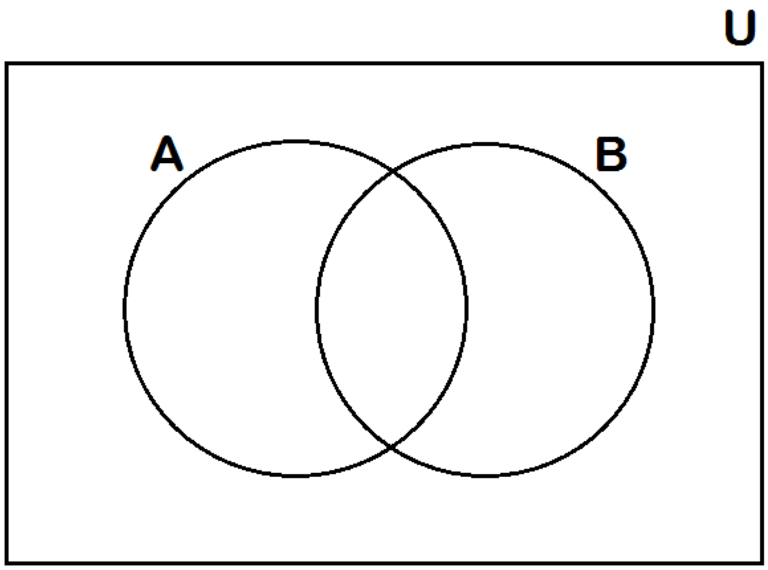
(a) $\varnothing, \{a\}, \{b\}, \{a, b\}$

(b) $\varnothing, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}$

(c) $\varnothing, \{\alpha\}, \{\beta\}, \{\gamma\}, \{\delta\}, \{\alpha, \beta\}, \{\alpha, \gamma\}, \{\alpha, \delta\}, \{\beta, \gamma\}, \{\beta, \delta\}, \{\gamma, \delta\},$
$\{\alpha, \beta, \gamma\}, \{\alpha, \beta, \delta\}, \{\alpha, \gamma, \delta\}, \{\beta, \gamma, \delta\}, \{\alpha, \beta, \gamma, \delta\}$

Venn Diagrams and Set Operations. Given sets, there are various operations we can perform with them. To see these, it can be useful to visualize these with Venn Diagrams. First, we imagine that all of the sets in our problem live in some *universal set*, which we will denote by U, that is, we will assume that all of our sets are subsets of U.

To illustrate the set operations, we will use both actual sets and Venn diagrams. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 2, 3, 4, 5\}$, and $B = \{3, 4, 5, 6, 7\}$. For the Venn diagram, we will shade in the relevant regions of this diagram:



We have the following definitions:

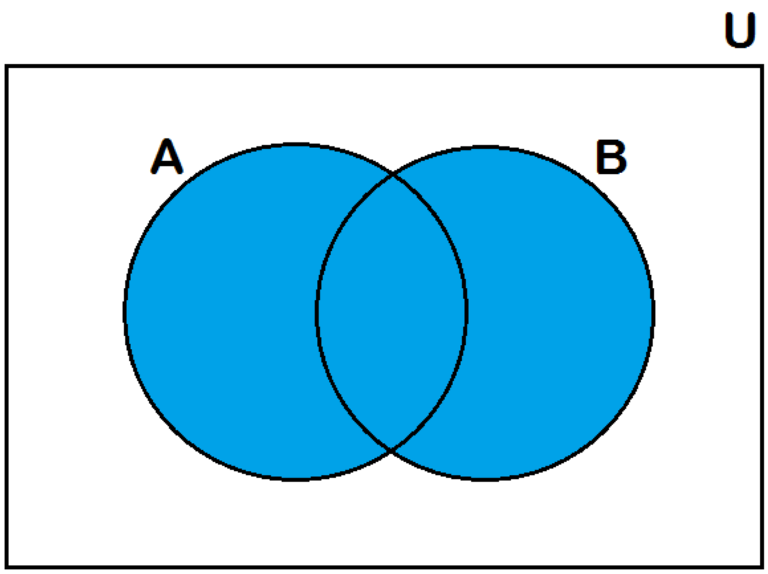
Definition 1 (Union)**.** *The* union *of two sets A and B is the new set, denoted $A \cup B$, which consists of all elements which are in A or in B.*

$$A \cup B = \{x | x \in A \text{ or } x \in B\}.$$

Using the sets above,

$$A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$$

and as a Venn Diagram



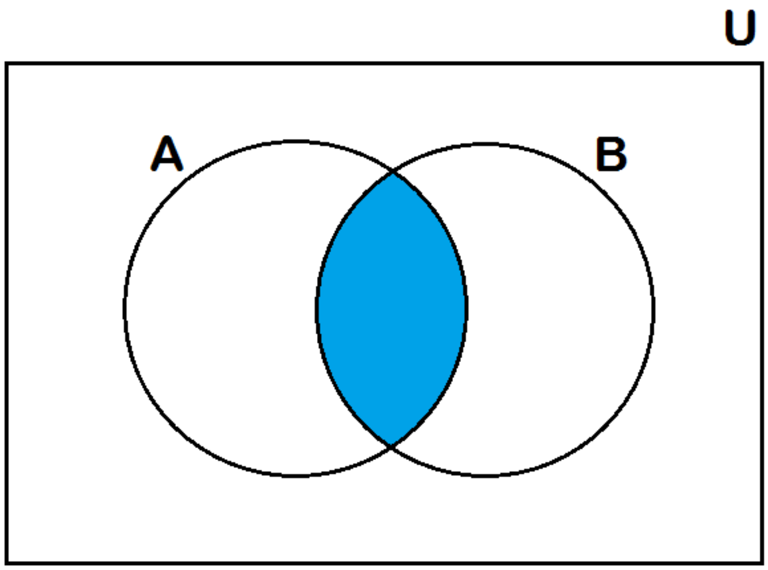
Definition 2 (Intersection)**.** *The* intersection *of two sets A and B is the new set, denoted $A \cap B$, which consists of all elements which are in A and in B.*

$$A \cap B = \{x | x \in A \text{ and } x \in B\}.$$

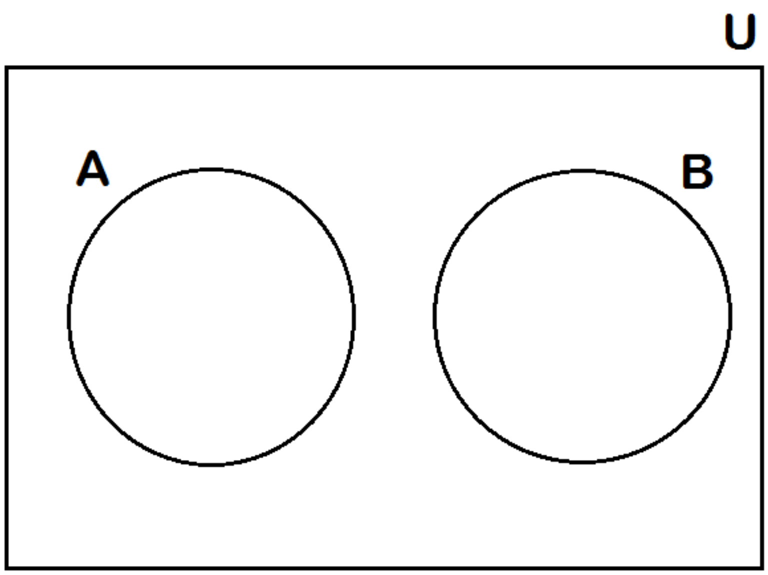
Using the sets above,

$$A \cap B = \{3, 4, 5\}$$

and as a Venn Diagram



In general, it is possible that two sets do not have any elements in common. For the moment, assume $B = \{7, 8, 9\}$, then $A \cap B = \varnothing$ and as a Venn diagram we have a picture like:



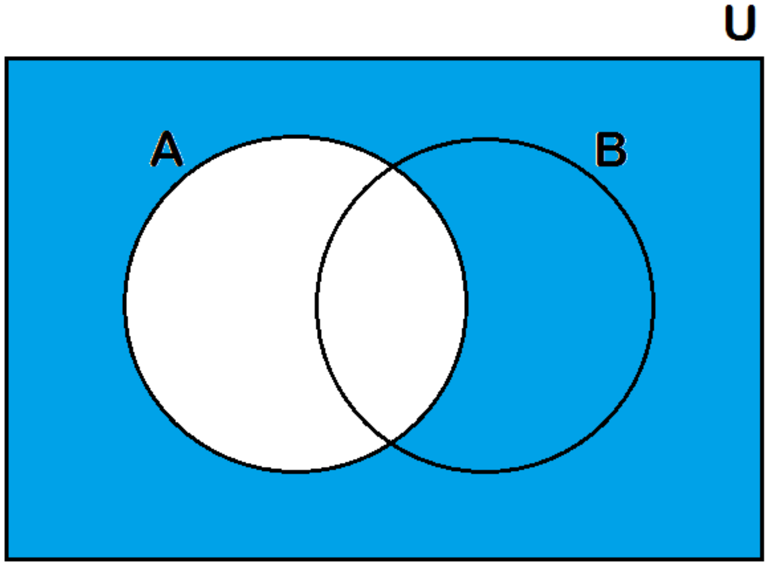
Definition 3 (Complement)**.** *The* complement *of a set A is the new set, denoted A', which consists of all elements which are in U, but not in A.*

$$A' = \{x \in U | x \notin A\}.$$

Using the sets above,

$$A' = \{6, 7, 8, 9\}$$

and as a Venn Diagram


U
A
B